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Semiglobal Alice Strings

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Abstract

We show that in certain theories with topologically trivial quotient space of spontaneously broken gauge symmetry there can exist topologically stable strings that carry non-abelian gauge flux. These objects result from the “accidental” global degeneracy of the vacuum which makes it topologically nontrivial. In particular, some models contain the semiglobal analog of the Alice strings.

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Introduction. Many particle physics models in the modern cosmological context lead to the formation of stable vacuum defects. Their origin has to do with the topology of the vacuum manifold V . When the first homotopy group $\pi_1(V)$ is nontrivial, the resulting defects are strings. If V is topologically equivalent to the quotient space G_l/H_l of the spontaneously broken gauge symmetry $G_l \rightarrow H_l$ (meaning that there is no additional global degeneracy in V), strings carry a topologically stable gauge flux which compensates the gradient of the Higgs field at spatial infinity [1]. Such strings are called local (or gauge) strings.

Due to the above, it is believed sometimes that the existence of topologically stable flux carrying strings can be recognised by examining the first homotopy group of the quotient space G_l/H_l . That is, the flux persists (topologically) only if $\pi_1(G_l/H_l) \neq 1$. This criterion is based on the assumption that even if the actual global structure of V is larger than G_l/H_l (so that $\pi_1(V) \neq 1$), at best the resulting defect can be global strings empty of flux. As we have shown recently [2] this assumption is false: topologically stable flux carrying strings can exist even if $\pi_1(G_l/H_l) = 1$, provided the global vacuum manifold V is topologically nontrivial. That is

$$\pi_1(G_{gl}/H_{gl}) \neq 1 \tag{1}$$

where G_{gl} (covering G_l) and H_{gl} (covering H_l) are the initial and final global symmetry groups respectively. In particular, as was shown in [2], such topologically stable flux carrying strings exist automatically in the $SU(2) \otimes U(1)$ electroweak model provided the theory contains at least one

spontaneously broken global $U(1)_{gl}$ factor under which the electroweak Higgs doublet(s) transforms nontrivially. In particular the role of $U(1)_{gl}$ can be played by R -symmetry, Peccei-Quinn or $(B - L)$ symmetry. The topologically stable flux carried by such strings is rather unusual: it does not compensate the logarithmic divergence of the Higgs gradient energy at spatial infinity. Therefore strings carry properties of both global and gauge strings, and in [2] we have called them “semiglobal”. The semiglobal strings should not be confused with the semilocal ones [3] for which the situation is just opposite: $\pi_1(G_l/H_l) \neq 1$ whereas $\pi_1(G_{gl}/H_{gl}) = 1$, and the flux is always topologically unstable (while it can be classically stable in a certain range of parameters). Semiglobal strings discussed in [2] are of Abelian nature, since their stability is due to the breaking of an Abelian $U(1)_{gl}$ factor that renders V topologically nontrivial.

In the present paper we investigate the existence of nonabelian semiglobal strings. In particular we show that certain realistic models contain the semiglobal analogs of “Alice” strings [4]. These strings are formed due to the breaking of “accidental” nonabelian global symmetries of the classical Higgs potential which satisfy (1). These symmetries are accidental in the sense that they are not respected by all (and in particular by gauge) interactions of the theory, but are automatically compatible with *all possible* gauge invariant terms of the classical Higgs potential. Therefore at the classical level they provide a topological nontrivial degeneracy of the vacuum leading to the formation of strings. The quantum corrections in general lift the accidental nonabelian degeneracy of the vacuum and make it topologically trivial. However the strength of this correction is controlled by the gauge

coupling constant so that they do not alter the classical stability of the flux in a wide range of parameters. The above consideration provides a crucial difference between abelian and nonabelian semiglobal strings: in contrast with nonabelian ones, the accidental abelian symmetries of the scalar potential are automatically respected by the gauge interaction and appear to be true global symmetries of the theory (unless broken by anomalies or by Yukawa interactions). Consequently, the abelian semiglobal strings remain topologically stable even after quantum corrections.

Topologically stable nonabelian flux. As an example consider the simplest model that gives rise to a gauge Alice string [4]. Such is the one with an $SO(3)$ -local symmetry spontaneously broken down to the semidirect product $U(1) \otimes Z_2$ by a real Higgs field Σ_{jk} ($j, k = 1, 2, 3$) in the 5-dimensional tensor representation and whose vacuum expectation value (VEV) has the form

$$\Sigma_0 = \text{diag}(1, 1, -2)\sigma \quad (2)$$

Along the path enclosing the straight infinite string the VEV of Σ winds by an $SO(3)$ group transformation

$$\Sigma(\theta) = U(\theta)\Sigma_0U(\theta) \quad (3)$$

interpolating between $Z_2 (= U(2\pi))$ and identity ($= U(0)$), where $\Delta\theta = 2\pi$ along the path. For instance, we can choose

$$U(\theta) = \exp(iT^1\theta/2) \quad (4)$$

where T^1 is the generator of rotations in the 2-3 plane. The string carries a

topologically stable magnetic flux of the gauge field A_μ^1 associated with the generator T^1 . The above string has the well known “Alice” property that the $U(1)$ -charge of the point particle gets flipped when the later is transported around the string. This is due to the fact that generator of Z_2 conjugates the $U(1)$ -charge.

Let us imagine now that there is an additional Higgs doublet field $\Phi = (\phi_1, \phi_2)$ in the spinorial representation, which breaks the gauge symmetry completely. We assume that the VEV-s of Φ and Σ are of the same order of magnitude. Then, the gauge symmetry is broken completely *at one and the same* stage and its quotient space is topologically trivial. So naively one does not expect formation of any flux carrying (or even global) stable defects in this model. However this assumption is false: at the classical level this model contains the *topologically* stable semiglobal Alice string. The key point is that the most general $SU(2)$ -symmetric renormalizable potential of the fields Σ and Φ is invariant under a larger global $G_{gl} = SO(3)_\Sigma \otimes SU(2)_\Phi$ symmetry of independent global transformations (the index indicates on which field the given group transformation acts). This is a consequence of the group-theoretical fact that the only possible (up to a quartic) $SU(2)$ -invariant constructed out of the one 5-dimensional and one spinorial representation is the trivial one $(Tr \Sigma^2)(\Phi^\dagger \Phi)$.

More importantly perhaps, G_{gl} is not just a symmetry of the Higgs potential only. In fact it is the actual global symmetry of the *full* Lagrangian on the arbitrarily fixed $\Phi = \text{constant}$ surfaces. This can be readily seen from the expression of the most general renormalizable $SU(2)$ -gauge invariant Lagrangian of Σ and Φ which has the following form:

$$\begin{aligned}
L = & \left[ig A_\mu^a \Phi^\dagger \frac{\sigma^a}{2} \partial_\mu \Phi + hc \right] + (\partial_\mu \Phi)^\dagger (\partial_\mu \Phi) + \frac{g^2}{4} (A_\mu^a A^{a\mu}) \Phi^\dagger \Phi + \\
& tr \left[\partial_\mu \Sigma - ig A_\mu^a [T^a, \Sigma] \right]^2 - \frac{1}{4} tr F_{\mu\nu} F^{\mu\nu} - V(\Sigma, \Phi)
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
V(\Sigma, \Phi) = & m_1^2 tr \Sigma^2 + h_1 (tr \Sigma^2)^2 + h_2 tr \Sigma^4 + m_2 tr \Sigma^3 \\
& + h_3 (tr \Sigma^2) (\Phi^\dagger \Phi) + m_3 (\Phi^\dagger \Phi) + h_4 (\Phi^\dagger \Phi)^2
\end{aligned} \tag{6}$$

and T^a and σ^a ($a = 1, 2, 3$) are the $SU(2)_l$ generators in the vectorial and spinorial representations respectively, and other indices are suppressed. Obviously all the terms in (5), except the first one, are invariant under the independent global transformations:

$$\Sigma \rightarrow U_\Sigma^{-1} \Sigma U_\Sigma ; \quad A_\mu \rightarrow U_\Sigma A_\mu ; \quad \Phi \rightarrow U_\Phi \Phi ; \tag{7}$$

where $U_\Sigma = \exp[i\theta^a T^a]$, $U_\Phi = \exp[i\beta^b \sigma^b]$, and θ^a ($a = 1, 2, 3$) and β^b ($b = 1, 2, 3$) are two arbitrary independent sets of group parameters, and $A_\mu = (A_\mu^1, A_\mu^2, A_\mu^3)$. This means that the above transformation is an exact symmetry of the action on arbitrary $\Phi = \text{constant}$ surfaces (where the first two terms are zero) and has an important impact on the vacuum structure.

The only obvious assumption we make about the structure of the Higgs potential is that it has a stable global minimum for VEVs of Σ and Φ being nonzero and comparable. It can be checked readily that the only possible nonzero configurations of the VEV of Σ in the minimum are those that can be

obtained from (2) by all possible U_Σ transformations. Of course the relative orientation of the VEVs Σ and Φ are arbitrary, which simply reflects the global (7) symmetry of the potential. We can fix one of the fields, say Φ , by a gauge transformation in the form $\Phi = (\phi, 0)$ with ϕ real and positive. Then the theory will be minimized by arbitrary

$$\Sigma = U_\Sigma^{-1} \Sigma_0 U_\Sigma \quad (8)$$

where Σ_0 is given in (2). Different group elements U_Σ bring us to the different points of the continuously degenerated vacuum. These points strictly speaking are not physically equivalent, since (7) is not a true symmetry of the theory. However this vacuum misalignment is rather unusual. *Different points of the vacuum (obtained by different U_Σ) correspond to one and the same pattern of the gauge symmetry breaking and to one and the same particle spectrum.* (Of course the mass eigenstates vary from point to point). This is closely related to the fact that (7) is the actual symmetry for $\phi = \text{constant}$. This implies further that the misalignment is not resolved by one loop radiative corrections and the one loop effective potential remains degenerate under (7) (recall that the one-loop correction depends only on the particle spectrum [5]).

Now let us turn to the topological structure of the vacuum. The global symmetry (7) that leaves invariant the classical (even one loop) potential is spontaneously broken down to $H_{gl} = (U(1) \otimes Z_2)_\Sigma$ which entirely belongs to $SO(3)_\Sigma$, and the vacuum manifold has a nontrivial first homotopy group. So there are topologically stable strings in this model around which Σ winds as in (3); the corresponding group transformation that interpolates between

identity and Z_2 now entirely belongs to $SO(3)_\Sigma$ and is given by (3). The most important point about these strings is that they carry a topologically stable gauge flux.

Consider again the closed path that encircles the string at infinity. Let us now, as before, fix the gauge with $\Phi = (\phi, 0)$ being real and positive everywhere. Finiteness of the energy forces ϕ to be constant along the path. In contrast Σ cannot stay constant due to topological reasons, since it has to wind by the U_Σ transformation that interpolates between 1 and Z_2 . We choose this transformation in the form

$$\Sigma = \exp(-iT^1\theta/2)\Sigma_0\exp(iT^1\theta/2) \quad (9)$$

where $0 < \theta(x_\mu) < 2\pi$ is a continuous function of the spatial point. Whatever the explicit form of the function θ is, it should change by $\Delta\theta = 2\pi$ encircling the string. That is, we have the topological constraint

$$\frac{1}{2\pi} \oint \partial_\mu \theta(x) dx^\mu = 1 \quad (10)$$

where integration is performed along the same path. Using expression (5), we find the following equation of motion for the gauge field A_μ^1 at spatial infinity:

$$A_\mu^1 = \frac{\partial_\mu \theta}{g} \left(1 + \frac{\phi^2}{72\sigma^2}\right)^{-1} \quad (11)$$

Now performing integration along the same closed path and using the topological constraint (10) we obtain the following expression for the gauge flux:

$$flux = \frac{2\pi}{g} \left(1 + \frac{\phi^2}{72\sigma^2}\right)^{-1} \quad (12)$$

which is clearly non-integer [2]. This makes semi-global strings perfect candidates for the Aharonov-Bohm type enhancements of the fermion-string scattering amplitude [6].

Matter Fields. The physical picture depends sensitively on how the fermions are coupled to the scalar fields of the theory, i.e. in their representation content under the gauge group. Let us discuss the possible realizations case by case.

1. *Imagine the fermion fields to be triplets under $SU(2)_l$* , in which case they are coupled to the field Σ . This is the conventional case of the Alice string discussed in the literature [4]. Since both the fermions and the gauge fields are triplets, they are rotated in the same manner as they go around the string and thus locally a “fermionic” observer will see no difference (including one loop effects). However, since the Z_2 generator reverses the signs of ‘electric’ charges, after the full rotation a ‘particle’ ends up as an ‘antiparticle’. Of course, in this case the ‘photon’ is not massless since $SU(2)_l$ is broken completely; it is just the lightest gauge boson.
2. *Fermions coupled to the doublet Φ* , by say left-handed fields forming doublets and the right-handed ones singlets under the gauge group. Now, since Φ remains constant as it travels around the string, the same will be true of the fermionic fields. On the other hand, the gauge fields, as we said, change (the eigenstates, not the eigenvalue), and thus,

in this case the physical consequences vary *locally* from one point to another, including the phenomenon of flavor changing.

3. *The fermions decoupled from both Σ and Φ .* In this case, the situation is basically equivalent to the one described in 2.

Discussion The simple models we have discussed play a direct role in the theories of family unification, along the lines discussed by us before [7].

Among the realistic models the ones that often exhibit nonabelian accidental degeneracy of the vacuum are supersymmetric GUTs. In these models the global degeneracy of the vacuum can result from two sources: the group structure of the Higgs sector (as in nonsupersymmetric case) and the supersymmetry. Due to the usual nonrenormalization theorem, supersymmetry can support the accidental degeneracy in the vacuum manifold even if this degeneracy is not respected by the gauge structure of the theory.

We will address those and related issues in detail in a future publication.

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